Course: Power Systems dynamics & Stability
EE — 105

Monday : III period

Wednesday : III Period

Thursday : II Period

Books:

1. Power system Stability & Control
Prabha Kundur [McGraw Hill]

2. Power system Dynamics, Stability & Control
K R Padiyar [BS Publications, Hyderabad]

3. Power system Control & Stability
P.M. Anderson & A.A. Fouad [Wiley India]

4. Elements of Power System Analysis
William D. Stevenson, Jr. [McGraw Hill]

5. Power system Dynamics and Stability
Peter W. Sauer, M.A. Pai, Joe H. Chow
[Wiley IEEE Press]

6. Power System Analysis
   Hadi Saadat - [McGraw Hill]

7. Power System Analysis -
   Kothari & Nagrath [McGraw Hill]

Papers :

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Power system stability :

A power system is highly nonlinear system and continuously experiences disturbances. Power system stability is a complex subject that has challenged the power system engineers.

The stability classes (classification) pertaining the present-day power systems (widely interconnected) are :

1. Angle stability
2. Voltage stability
3. Frequency stability

Angle stability :

This is concerned with the ability of interconnected synchronous machines to remain in synchronism after being subjected to a perturbation.

To understand angle stability we have to understand the rotor dynamics of a synchronous m/c.

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The equation governing the motion of the rotor of a synchronous machine is based on the elementary principle in dynamics which states that accelerating torque is product of moment of inertia of the rotor times its angular acceleration

For a synchronous generator we can write

$$J \frac{d^2 \theta_m}{dt^2} = T_a = T_m - T_e \quad N\text{-}m$$

where

J = the moment of inertia of the rotor masses, $kg\text{-}m^2$

θ_m = the angular displacement of the rotor with respect to a stationary axis, mechanical radians

t = time in seconds

T_m = the mechanical or shaft torque supplied by the prime mover less retarding torque due to rotational losses, N-m.

T_e = electromagnetic torque, N-m.

T_a = net accelerating torque, N-m.

T_m is the torque which tends to accelerate the rotor in the positive θ_m direction, as shown in figure.

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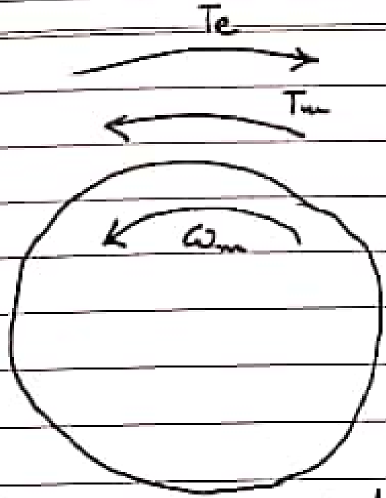


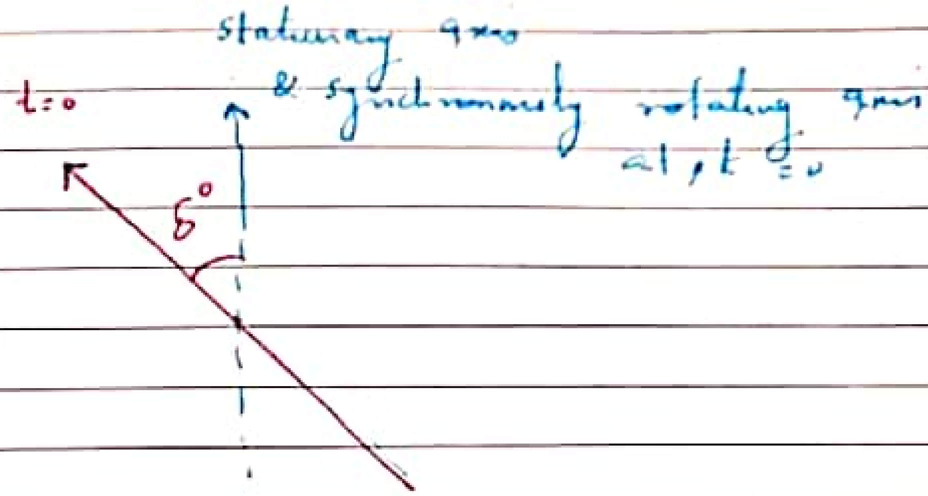
Under steady state conditions, T_m and T_e are equal and the accelerating torque T_a is zero. In this case there is no acceleration or deceleration of the rotor and the resultant speed is the synchronous speed. The m/c will be in synchronism with the other m/cs in the power system.

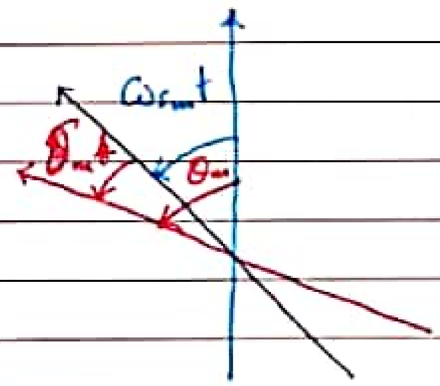
Since θ_m is measured with respect to a stationary reference axis, therefore it continuously increases with time even at constant synchronous speed.

Rotor speed relative to synchronous speed is of interest, it is more convenient to measure the rotor angular position with respect to a reference axis which rotates at synchronous speed. Therefore we define

$$\theta_m = \omega_{sm} t + \delta_m$$

Where ω_{sm} is the synchronous speed and δ_m is angular position of the rotor in mechanical radians from the synchronously rotating reference axis





$$\theta_m = \omega_{sm} t + \delta_m$$

$$\delta_m(t) = (\omega_{rm} t - \omega_{sm} t) + \delta_m^\circ$$

$$\boxed{\dot{\delta}_m(t) = (\omega_{rm} - \omega_{sm})}$$

$$\delta_m(0) = \delta^\circ$$

$$[\dot{\delta} = (\omega_r - \omega_s)]$$

$$\frac{d\theta_m}{dt} = \omega_s + \frac{d}{dt}(\delta_m)$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

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Substituting (2) in (1)

$$J \frac{d^2 \delta_m}{dt^2} = T_m - T_e$$

$$J \omega_{rm} \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{S_{rated}};$$

$$= \frac{\text{Stored kinetic energy in MJ at syn. speed}}{\text{machine rating MVA}}$$

$$J = \frac{2 H S_{machine}}{\omega_{sm}^2}$$

$$\therefore \left(\frac{\omega_{rm}}{\omega_{sm}^2}\right) \frac{2 H S_{rated}}{} \cdot \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

$$\left(\frac{\omega_{rm}}{\omega_{sm}^2}\right) 2H \frac{d^2 \delta_m}{dt^2} = P_m (p.u) - P_e (p.u)$$

It is reasonable to assume

$\omega_{rm} = \omega_{sm}$

even under transient conditions

$$\therefore \frac{2H}{\omega_{sm}} \frac{d^2 \delta_m}{dt^2} = P_m(pu) - P_e(pu).$$

In electrical quantities $\left[\delta = \frac{P}{2}\delta_m \quad \omega_s = \frac{P}{2}\omega_{sm} \right]$

$$\frac{2H}{\omega_s} \frac{d^2 \delta_e}{dt^2} = P_m - P_e,$$

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e.$$

$\delta \rightarrow$ elect. rad.

$$\frac{H}{180 f} \cdot \frac{d^2 \delta}{dt^2} = P_m - P_e.$$

Swing equation. δ in elect degrees.

The solution of this equation gives δ as a function of t. A graph of the solution is known as swing curve. Inspection of the swing curves of all the machines of a system will show whether the machines will remain in synchronism after a disturbance.

We note that swing equation is a second-order differential equation which can be written as the two, first-order differential equations

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$x_1 = \delta$$

$$x_2 = \dot{\delta}$$

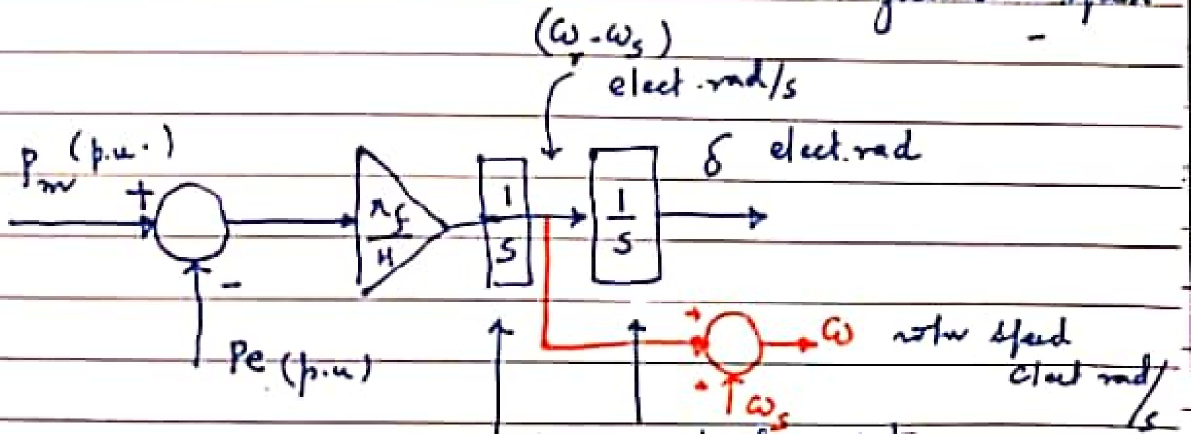
$$\dot{x}_1 = x_2$$

$$\frac{H}{\pi f}\dot{x}_2 = P_m - P_e$$

$$\boxed{\dot{x}_2 = \frac{\pi f}{H}\left[P_m - P_e\right]}$$

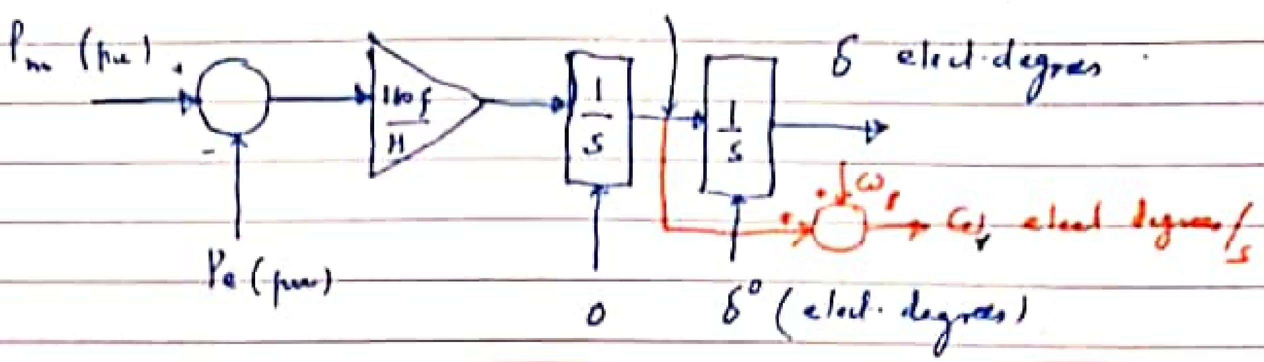
$x_1 = \delta$ elect rad

$x_2 = \omega_r - \omega_s$ ← relative speed between rotor speed & synchronous speed



Initial condition 0 $(\omega - \omega_s)$ ↑ If rotor initially rotates at synchronous speed

initial conditions δ^0



Representation of a multi-machine power plant

In a stability study for a large system with many machines geographically dispersed over a wide area, it is desirable to minimize the number of swing equations to be solved. This can be done if the transmission line fault, or other disturbance of the system affects the machines within a power plant so that their rotors swing together. In such cases, the machines within the plant can be combined into a single equivalent m/c and only swing equation needs to be written.

Consider a power plant with two generators connected to the same bus which is electrically remote from the network disturbance.

The swing equations on the common system base are

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{m_1} - P_{e_1} \quad \text{per unit}$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{m_2} - P_{e_2} \quad \text{per unit}$$

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Adding the equations together and denoting δ_1 and δ_2 by δ (Since the rotors swing together), we obtain

$$\frac{H_1+H_2}{\pi f}\frac{d^2\delta}{dt^2} = (P_{m_1}+P_{m_2}) - (P_{e_1}+P_{e_2})$$

or $$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

Where $H = H_1 + H_2$ & $P_m = P_{m_1} + P_{m_2}$, $P_e = P_{e_1} + P_{e_2}$

Machines which swing together are called coherent machines. It is noted that when δ & ω_s and δ are expressed in electrical degrees or radians, the swing equations for coherent machines can be combined together even though the rated speeds are different.

In general if the plant has n machines with inertia constants $H_1, H_2 \ldots H_n$ and ratings $S_{mach1}, S_{mach2} \ldots S_{machn}$

then $$H = \frac{H_1 \times S_{mach1} + H_2 \times S_{mach2} + \ldots + H_n \times S_{machn}}{S_{system}}$$

Example: Two 60 Hz generating units operate in parallel within the same power plant and have the following ratings:

Unit 1: 500 MVA, 0.85 pf, 20 kV, 3600 r/min.

Unit 2: 1333 MVA, 0.9 pf, 22 kV, 1800 r/min

$H_1 = 4.8$ MJ/MVA and $H_2 = 3.27$ MJ/MVA

Calculate equivalent H constant for two units on 100 MVA base.

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Sol: $\frac{4.8 \times 500 + 3.27 \times 1333}{100} = 67.59$ MJ/MVA

Representation for a pair of non-coherent machines in a power system

For m/c 1 the Swing equation is :

$$\frac{H_1}{\pi f} \frac{d^2\delta_1}{dt^2} = P_{m_1} - P_{e_1}$$

& for m/c 2 we have

$$\frac{H_2}{\pi f} \frac{d^2\delta_2}{dt^2} = P_{m_2} - P_{e_2}$$

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{\pi f [P_{m_1} - P_e]}{H_1} - \frac{\pi f [P_{m_2} - P_{e_2}]}{H_2}$$

Let $\delta_1 - \delta_2 = \delta$

& multiply each sides by $\frac{H_1 H_2}{H_1 + H_2}$

we have

$$\left(\frac{H_1 H_2}{H_1 + H_2}\right) \frac{d^2\delta}{dt^2} = \pi f \left[\frac{H_2 P_{m_1} - H_2 P_{e_1}}{H_1 + H_2}\right] - \pi f \left[\frac{H_1 P_{m_2} - H_1 P_{e_2}}{H_1 + H_2}\right]$$

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or $\frac{1}{\pi f}\left(\frac{H_1 H_2}{H_1+H_2}\right)\frac{d^2\delta}{dt^2} = \left(\frac{P_{m_1}H_2 - P_{m_2}H_1}{H_1+H_2}\right) - \left(\frac{P_{e_1}H_2 - P_{e_2}H_1}{H_1+H_2}\right)$ — (Eq

or $\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_{meq} - P_{eeq}$

Where

$$H_{eq} = \frac{H_1 * H_2}{H_1 + H_2}$$

$$P_{meq} = \frac{P_{m_1}H_2 - P_{m_2}H_1}{H_1 + H_2}$$

$$P_{eq} = \frac{P_{e_1}H_2 - P_{e_2}H_1}{H_1 + H_2}$$

Eq is convenient for analysing two m/c problems such as:

i) Generator - infinite bus

ii) Synchronous generator and a synchronous motor connected by a network of pure reactances.

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1) Generator – infinite bus*

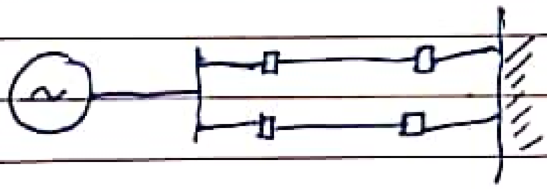
If we convert the second synchronous machine into an infinite bus by letting H_2 tend to infinity, then we have

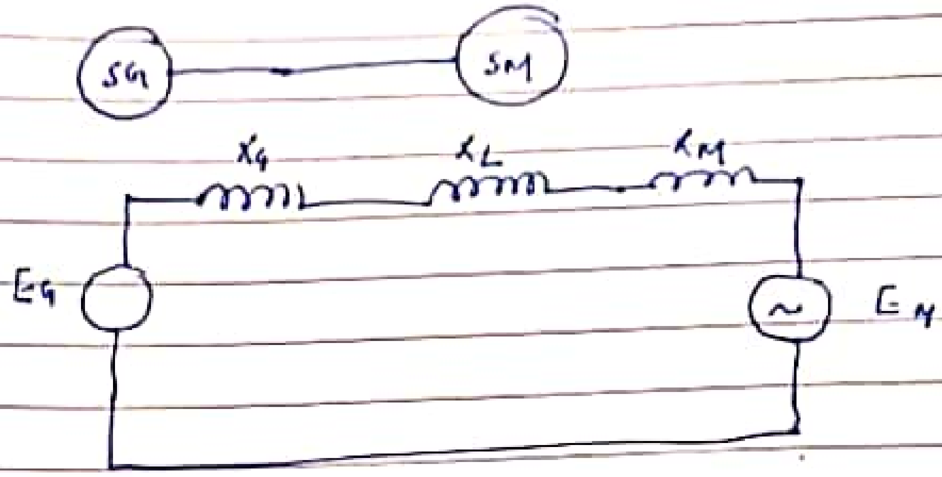
$$\lim_{H_2 \to \infty} \frac{1}{\pi f}\left(\frac{H_1}{\frac{H_1}{H_2}+1}\right)\frac{d^2\delta}{dt^2} = \left[\frac{P_{m_1} - \frac{P_{m_2}\cdot H_1}{H_2}}{\frac{H_1}{H_2}+\frac{H_2}{H_2}}\right]$$

$$- \left[\frac{Pe_1 - Pe_1 H_1/H_2}{1+\frac{H_1}{H_2}}\right]$$

$$\boxed{\frac{H_1}{\pi f}\frac{d^2\delta_{12}}{dt^2} = P_{m_1} - Pe_1}$$

* An infinite bus may be considered for stability purposes as a bus at which is located a machine of constant internal voltage, having zero impedance and infinite inertia.





A noteworthy application of eq. — concerns a two-m/c system having only one generator (machine 1) and a synchronous motor (machine 2) connected by a network of pure reactances. Whatever change occurs in the generator output is thus absorbed by the motor and we can write.

$$P_{m1} = -P_{m2} = P_m$$

$$P_{e1} = -P_{e2} = P_e$$

$$\therefore P_{meq} = \frac{P_m H_2 + P_m H_1}{H_1 + H_2} = P_m$$

$$\& \quad P_{eq} = \frac{P_e H_2 + P_e H_1}{H_1 + H_2} = P_e$$

Swing equation is

$$\frac{H_{eq}}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$